

$B \rightarrow \chi_{c1}(1P, 2P)K$ decays in QCD factorization and X(3872)

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$B \rightarrow \chi_{c1}(1P, 2P)K$ decays are studied in QCD factorization by treating charmonia as nonrelativistic bound states. No infrared divergences exist in the vertex corrections, while the logarithmic end-point singularities in the hard spectator corrections can be regularized by a momentum cutoff. Within certain uncertainties we find that the $B \rightarrow \chi_{c1}(2P)K$ decay rate can be comparable to $B \rightarrow \chi_{c1}(1P)K$, and get $Br(B^0 \rightarrow \chi'_{c1} K^0) = Br(B^+ \rightarrow \chi'_{c1} K^+) \approx 2 \times 10^{-4}$. This might imply a possible interpretation for the newly discovered X(3872) that this state has a dominant $J^{PC} = 1^{++}(2P)$ $c\bar{c}$ component but mixed with a substantial $D^0 \bar{D}^{*0} + D^{*0} \bar{D}^0$ continuum component.

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The naively factorizable decay [1] $B \rightarrow \chi_{c1}K$ was studied [2] in the QCD factorization approach [3] in which the nonfactorizable vertex and spectator corrections were also estimated, but the numerical results were four times smaller than experimental data. Recently, these decays were also studied in the PQCD approach [4]. In both the above approaches, light-cone distribution amplitudes (LCDAs) were used to describe χ_{c1} . As argued in Ref. [5], a more appropriate description of charmonium is the nonrelativistic (NR) wave functions which can be expanded in terms of the relative momentum q between charm and anticharm quarks. This argument is based on the nonrelativistic nature of heavy quarkonium [7]. With careful studies, we find that the two descriptions (i.e. LCDAs and NR) are equivalent for the S-wave charmonium states (see, e.g. [6]), but in the case of P-wave states the light-cone descriptions lose some important contributions in the leading-twist approximation. This is not surprising since q can be neglected in S-wave states, but cannot be neglected for P-wave states even in leading order approximation.

On the phenomenological hand, the study of $B \rightarrow \chi_{c1}(2P)K$ may help clarify the nature of the recently discovered resonance X(3872) [8], since the measurements for X(3872) favor $J^{PC} = 1^{++}$ [9] and hence $\chi_{c1}(2P)$ becomes one of the possible assignments for it. On the other hand, aside from the conventional charmonium [10, 11], a loosely bound S-wave molecule of $D^0 \bar{D}^{*0} + D^{*0} \bar{D}^0$ has been suggested for X(3872) [12, 14].

Motivated by the above considerations, in this paper we study the decays $B \rightarrow \chi_{c1}(1P, 2P)K$ within the framework of QCD factorization by treating the charmonia $\chi_{c1}(1P, 2P)$ as nonrelativistic bound states with m_c/m_b taken to be a fixed value in the heavy b quark limit. We will estimate the production rate of $\chi_{c1}(2P)$ and argue that the X(3872) may be dominated by the $\chi_{c1}(2P)$ charmonium but mixed with some $D^0 \bar{D}^{*0} + D^{*0} \bar{D}^0$ continuum component.

In the non-relativistic bound-state picture, charmonium can be described by the color-singlet NR wave function. Let p be the total momentum of the charmonium

and $2q$ be the relative momentum between c and \bar{c} quarks, then $v^2 \sim 4q^2/p^2 \sim 0.25$ can be treated as a small expansion parameter [7]. For P-wave charmonium χ_{c1} , because the wave function at the origin $\mathcal{R}_P(0)=0$, which corresponds to the zeroth order in q , we must expand the amplitude to first order in q . Thus we have

$$\mathcal{M}(B \rightarrow \chi_{c1}K) = \sum_{L_z, S_z} \langle 1L_z; 1S_z | 1J_z \rangle \int \frac{d^4q}{(2\pi)^3} q_\alpha \delta(q^0) \times \psi_{1M}^*(q) \text{Tr}[\mathcal{O}^\alpha(0) P_{1S_z}(p, 0) + \mathcal{O}(0) P_{1S_z}^\alpha(p, 0)], \quad (1)$$

where $\mathcal{O}(q)$ represent the rest of the decay amplitudes and $P_{1S_z}(p, q)$ is the spin-triplet projection operator, and \mathcal{O}^α , P^α stand for the derivatives of \mathcal{O} , P with respect to the relative momentum q_α [5]. The amplitudes $\mathcal{O}(q)$ can be further factorized as product of $B \rightarrow K$ form factors and hard kernel or as the convolution of a hard kernel with light-cone wave functions of B meson and K meson, within QCD factorization approach.

After q^0 is integrated out, the integral in Eq. (1) is proportional to the derivative of the P-wave wave function at the origin by

$$\int \frac{d^3q}{(2\pi)^3} q^\alpha \psi_{1M}^*(q) = -i\varepsilon^{*\alpha}(L_z) \sqrt{\frac{3}{4\pi}} \mathcal{R}'_P(0), \quad (2)$$

where $\varepsilon^\alpha(L_z)$ is the polarization vector of an angular momentum-1 system and the value of $\mathcal{R}'_P(0)$ for charmonia can be found in, e.g., Ref. [15].

In contrast to the NR description of χ_{c1} , the K-meson is described by LCDAs [3]:

$$\langle K(p') | \bar{s}_\beta(z_2) d_\alpha(z_1) | 0 \rangle = \frac{if_K}{4} \int_0^1 dx e^{i(y p' \cdot z_2 + \bar{y} p' \cdot z_1)} \left\{ \not{p}' \gamma_5 \phi_K(y) \right\}_{\alpha\beta}, \quad (3)$$

where y and $\bar{y} = 1 - y$ are the momentum fractions of the s and \bar{d} quarks inside the K meson respectively, and $\phi_K(x)$ is the leading twist LCDA of K-meson. The masses of light quarks and K meson are neglected in heavy quark limit.

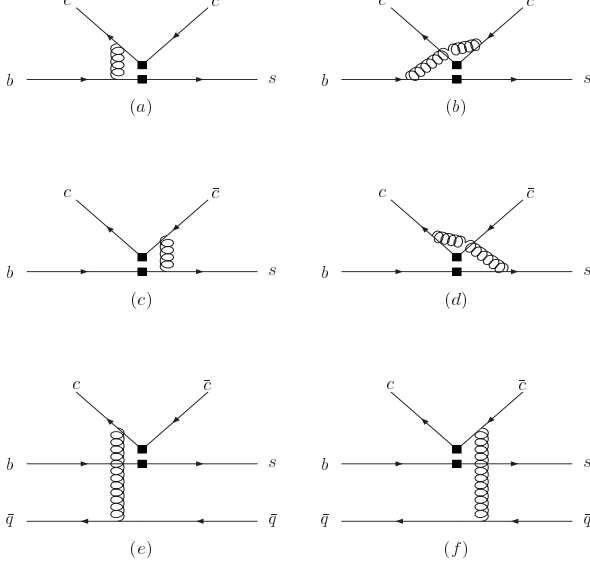


FIG. 1: Feynman diagrams for vertex and spectator corrections to $B \rightarrow \chi_{c0} K$.

The effective Hamiltonian for $B \rightarrow \chi_{c1} K$ reads [16]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(V_{cb} V_{cs}^* (C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2) - V_{tb} V_{ts}^* \sum_{i=3}^6 C_i \mathcal{O}_i \right), \quad (4)$$

where G_F is the Fermi constant, C_i are the Wilson coefficients and $V_{q_1 q_2}$ are the CKM matrix elements. The relevant 4-fermion operators \mathcal{O}_i can be found in [5].

According to [3] all nonfactorizable corrections are due to Fig.1. These corrections, with operators \mathcal{O}_i inserted, contribute to the amplitude $\mathcal{O}(q)$ in Eq. (1), where the external lines of charm and anti-charm quarks have been truncated. Taking nonfactorizable corrections in Fig.1 into account, the decay amplitude for $B \rightarrow \chi_{c1} K$ in QCD factorization is written as

$$i\mathcal{M} = \frac{G_F}{\sqrt{2}} \left[V_{cb} V_{cs}^* a_2 - V_{tb} V_{ts}^* (a_3 - a_5) \right] \times 12i \sqrt{\frac{2}{\pi M}} \mathcal{R}'_P(0) \epsilon^* \cdot p_B F_1(M^2), \quad (5)$$

where ϵ is polarization vector of χ_{c1} [17]. Here F_1 is the $B \rightarrow K$ form factor and we have used the relation $F_0(M^2)/F_1(M^2) = 1 - z$ [18], with $z = M^2/m_B^2 \approx 4m_c^2/m_b^2$ and M is the mass of χ_{c1} , to simplify the structure of (5).

The coefficients a_i ($i = 2, 3, 5$) in the naive dimension

regularization(NDR) scheme are given by

$$\begin{aligned} a_2 &= C_2 + \frac{C_1}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_1 \left(-18 + 12 \ln \frac{m_b}{\mu} + f_I + f_{II} \right), \\ a_3 &= C_3 + \frac{C_4}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_4 \left(-18 + 12 \ln \frac{m_b}{\mu} + f_I + f_{II} \right), \\ a_5 &= C_5 + \frac{C_6}{N_c} - \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_6 \left(-6 + 12 \ln \frac{m_b}{\mu} + f_I + f_{II} \right), \end{aligned} \quad (6)$$

where $C_F = (N_c^2 - 1)/(2N_c)$ and μ is the QCD renormalization scale.

The function f_I is calculated from the four vertex correction diagrams (a, b, c, d) in Fig.1 and reads

$$f_I = \frac{2z}{2-z} - \frac{4z \log(4)}{2-z} - \frac{4z^2 \log(z)}{(1-z)(2-z)} + \frac{4(3-2z)(1-z)(\log(1-z) - i\pi)}{(2-z)^2}, \quad (7)$$

We find that the infrared divergences are canceled between diagrams (a) and (b), (c) and (d) respectively in Fig.1. On the other hand, this function is different from that in Eq. (11) of Ref. [2] even when a nonrelativistic limit wave function $\phi_{\chi_{c1}}^{NR}(u) = \delta(u - 1/2)$ is adopted, as we have mentioned.

For the two spectator correction diagrams (e, f) in Fig.1, the off-shellness of the gluon is natural to be associated with a scale $\mu_h \sim \sqrt{m_b \Lambda_{\text{QCD}}}$, rather than $\mu_h \sim m_b$. Following Ref. [3], we choose $\mu = \sqrt{m_b \Lambda_h} \approx 1.4$ GeV with $\Lambda_h = 0.5$ GeV in calculating the hard spectator function f_{II} and then, in the leading twist approximation, we get

$$f_{II} = \frac{\alpha_s(\mu_h) C_i(\mu_h)}{\alpha_s(\mu) C_i(\mu)} \frac{8\pi^2}{N_c} \frac{f_K f_B}{F_1(M^2) m_B^2} \frac{1}{1-z} \times \int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \int_0^1 dy \frac{\phi_K(y)}{y} \left[1 + \frac{z}{y(1-z)} \right], \quad (8)$$

where ξ is the momentum fraction of the spectator quark in the B meson and $C_i(\mu_h)$ ($i = 1, 4, 6$) are the NLO Wilson coefficients which can be evaluated by the renormalization group approach [16].

The spectator contribution depends on the wave function ϕ_B through the integral

$$\int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \equiv \frac{m_B}{\lambda_B}. \quad (9)$$

Since $\phi_B(\xi)$ is appreciable only for ξ of order Λ_{QCD}/m_B , λ_B is of order Λ_{QCD} . We will choose $\lambda_B \approx 300$ MeV in the numerical calculations [3].

If we choose the asymptotic form of the K meson twist-2 LCDA, $\phi_K(y) = 6y(1-y)$, we can find logarithmic endpoint singularities in Eq. (8) just like that in Ref. [2], and we parameterize it in a simple way,

$$\int \frac{dy}{y} = \ln \frac{m_B}{\Lambda_h} \approx 2.4. \quad (10)$$

	a_2	a_3	a_5
$\chi_{c1}(3511)$	0.199-0.051i	0.000+0.002i	0.004-0.002i
$\chi'_{c1}(3953)$	0.247-0.042i	-0.002+0.001i	0.007-0.002i
$\chi'_{c1}(3872)$	0.236-0.044i	-0.002+0.001i	0.006-0.002i

TABLE I: The coefficients a_i of $B \rightarrow \chi_{c1}(1P, 2P)K$ with different choices of $M_{\chi'_{c1}}$.

The mass of $\chi_{c1}(1P)$ $M_{\chi_{c1}} = 3.511$ GeV is known, but the mass of the missing charmonium $\chi_{c1}(2P)$ has to be estimated by, say, potential models. We choose $M_{\chi'_{c1}} = 3.953$ GeV following Ref. [19]. Then the form factor $F_1(M^2)$ can be determined by light-cone sum rules [20],

$$F_1(M_{\chi_{c1}}^2) = 0.80, \quad F_1(M_{\chi'_{c1}}^2) = 1.14. \quad (11)$$

We also choose $M_{\chi'_{c1}} = 3.872$ GeV and $F_1(M_{\chi_{c1}}^2) = 1.06$ to study if the $X(3872)$ behaves like a $\chi_{c1}(2P)$ in their b-production processes.

For numerical analysis, we use the following input parameters :

$$\begin{aligned} m_b &= 4.8 \text{ GeV}, m_B = 5.28 \text{ GeV}, f_K = 160 \text{ MeV}, \\ f_B &= 216 \text{ MeV} [21], \mathcal{R}'_{1P}(0) = \mathcal{R}'_{2P}(0) = \sqrt{0.1} \text{ GeV}^{5/2}, \\ C_1(\mu) &= 1.21(1.082), C_2(\mu) = -0.40(-0.185), \\ C_3(\mu) &= 0.03(0.014), C_4(\mu) = -0.05(-0.035), \\ C_5(\mu) &= 0.01(0.009), C_6(\mu) = -0.07(-0.041), \\ \alpha_s(\mu) &= 0.35(0.22). \end{aligned} \quad (12)$$

In (12) the μ -dependent quantities at $\mu_h = 1.4$ GeV ($\mu = 4.4$ GeV) are shown without (with) parentheses.

Using the above inputs, we get the results of coefficients a_i which are listed in Table. I. With the help of these coefficients a_i , we calculate the decay branching ratios of decays $B \rightarrow \chi_{c1}(1P, 2P)K$ with two different choices of $M_{\chi'_{c1}}$ and get

$$\begin{aligned} \text{Br}(B^0 \rightarrow \chi_{c1}(3511)K^0) &= 1.79 \times 10^{-4}, \\ \text{Br}(B^0 \rightarrow \chi'_{c1}(3953)K^0) &= 1.81 \times 10^{-4}, \\ \text{Br}(B^0 \rightarrow \chi'_{c1}(3872)K^0) &= 1.78 \times 10^{-4}. \end{aligned} \quad (13)$$

Our prediction of $\text{Br}(B^0 \rightarrow \chi_{c1}(3511)K^0)$ is about 2 times larger than that in [2], although it is still about two times smaller than the recent data [22]. The difference between the theoretical predictions and experimental data may not be as serious as it looks like if we take into account the following uncertainties: (i) We have used a moderate value of $\mathcal{R}'_{1P}(0)$ predicted by different potential models [15] in our calculation, and a larger value of $\mathcal{R}'_{1P}(0)$ may enhance our prediction in Eq. (13) significantly. (ii) In evaluation of f_{1I} , we only use the leading twist LCDAs of K-meson, and large uncertainties will arise from the chirally enhanced higher twist effects [18]. (iii) Since the squared velocity v^2 of the charm quark in charmonium is about 0.25-0.30, the relativistic corrections may be important for these decays.

Note that although the form factor in (11) and the coefficient a_2 in Table. I increase evidently as the charmonium mass increases, the decreased phase space and kinematic factors in (5) will make a balance, and result in similar decay branching ratios in the charmonium mass region 3.51-3.95 GeV, as shown in (13). If we neglect the order α_s corrections (i.e., in the naive factorization [1]), the ratios among these three branching fractions in (13) would become 1 : 0.74 : 0.69. As estimated in (13), the branching ratios for $\chi_{c1}(2P)$ are

$$\begin{aligned} \text{Br}(B^0 \rightarrow \chi'_{c1}K^0) &\approx 2 \times 10^{-4}, \\ \text{Br}(B^+ \rightarrow \chi'_{c1}K^+) &= \text{Br}(B^0 \rightarrow \chi'_{c1}K^0). \end{aligned} \quad (14)$$

Comparing Eq. (14) with the measured channel of the $X(3872)$ [8]:

$$\begin{aligned} \text{Br}(B^+ \rightarrow XK^+) \times \mathcal{B}_X &= (1.3 \pm 0.3) \times 10^{-5}, \quad (15) \\ \mathcal{B}_X &\equiv \text{Br}(X \rightarrow J/\psi \pi^+ \pi^-), \end{aligned}$$

we see that the produced $X(3872)$ looks like the $\chi_{c1}(2P)$ if \mathcal{B}_X is sufficient small, say, $3 \sim 7\%$. A similar conclusion has recently been obtained in a comprehensive analysis of $X(3872)$ production at the Tevatron and B-factories [23]. On the other hand, if $X(3872)$ is a loosely bound S-wave molecule of $D^0 \bar{D}^{*0} / D^{*0} \bar{D}^0$ [12, 13], a model calculation gives a smaller rate [14] compared with Eq. (14):

$$\text{Br}(B^+ \rightarrow XK^+) = (0.07 \sim 1) \times 10^{-4}, \quad (16)$$

which requires a larger $\mathcal{B}_X > 10\%$ to be consistent with the experimental data (15). They also predict:

$$\text{Br}(B^0 \rightarrow X(3872)K^0) < 0.1 \text{Br}(B^+ \rightarrow X(3872)K^+). \quad (17)$$

So the measurement of \mathcal{B}_X and $\text{Br}(B^0 \rightarrow X(3872)K^0)$ is very helpful to identify the nature of $X(3872)$.

Recently, a preliminary result for a new decay mode $X \rightarrow D^0 \bar{D}^0 \pi^0$ was found by Belle[24]:

$$\begin{aligned} \text{Br}(B \rightarrow XK) \times \text{Br}(X \rightarrow D^0 \bar{D}^0 \pi^0) \\ = (2.2 \pm 0.7 \pm 0.4) \times 10^{-4}. \end{aligned} \quad (18)$$

Eq. (18) implies that $\mathcal{B}_X < 10\%$, if it can be confirmed by further measurements. This would disfavor the suggestion that the $X(3872)$ is a loosely bound S-wave molecule of $D^0 \bar{D}^{*0} / D^{*0} \bar{D}^0$ with predictions of both decay[13] and production[14].

The above discussions about the $X(3872)$ is based on the assumption that the $X(3872)$ is a pure charmonium $\chi_{c1}(2P)$ state. But this cannot be the case due to the coupled channel effects and $X(3872)$ being in extremely close proximity to the $D^0 \bar{D}^{*0} / D^{*0} \bar{D}^0$ threshold. Perhaps a more realistic model for the $X(3872)$ (for further discussions see [25]) is that the $X(3872)$ has a dominant $J^{PC} = 1^{++}(2P)$ $c\bar{c}$ component which is mixed with a substantial real $D^0 \bar{D}^{*0} / D^{*0} \bar{D}^0$ continuum component (the $D^+ \bar{D}^{*-} / D^{*-} \bar{D}^+$ continuum component is kinematically forbidden to be mixed in $X(3872)$ and it is the $u-d$

quark mass difference that causes this isospin violation). Thus $X(3872)$ will have the following features. (1) The production of $X(3872)$ in B meson decays is mainly due to the $J^{PC} = 1^{++}(2P)$ $c\bar{c}$ component, as discussed above. The production of $X(3872)$ at the Tevatron is also due to this $c\bar{c}$ component and associated higher Fock states containing the color-octet $c\bar{c}$ pair and soft gluons. As was argued [11] for the prompt charmonium production that cross sections of D-wave charmonia (which were suggested as a tentative candidates for $X(3872)$ in [11]) could be as large as J/ψ or $\psi(2S)$ due to the color-octet mechanism, the P-wave ($2P$) charmonium could also have the comparable production rate to J/ψ or $\psi(2S)$. But this does not seem to be obvious for a loosely bound S-wave molecule of $D^0\bar{D}^{*0}/D^{*0}\bar{D}^0$. (2) On the other hand, the $D^0\bar{D}^{*0}/D^{*0}\bar{D}^0$ continuum component in $X(3872)$ will be mainly in charge of the hadronic decays of $X(3872)$ into $D^0\bar{D}^{*0}/D^{*0}\bar{D}^0$ or $D^0\bar{D}^0\pi^0$ as well as $J/\psi\rho^0$ and $J/\psi\omega$. The latter two decay modes ($J/\psi\rho^0$ and $J/\psi\omega$) may come from the first decay mode $D^0\bar{D}^{*0}/D^{*0}\bar{D}^0$ and a subsequent rescattering final state interaction and therefore have the same decay amplitudes [$A(J/\psi\rho^0)=A(J/\psi\omega)$] that are smaller than the first decay mode amplitude. (3) A substantial $D^0\bar{D}^{*0}/D^{*0}\bar{D}^0$ continuum component in $X(3872)$ may reduce the production rates in Eq. (14), and will also reduce the $X(3872) \rightarrow J/\psi\gamma$ decay width, which can be as small as 11 KeV [10] (note that this 2P-1S E1 transition is sensitive to the model details, see, e.g. [13]). This is much smaller than the hadronic decay widths. But a large rate for $\chi_{c1}(2P) \rightarrow \gamma\psi(2S)=60-100$ KeV will be expected. These qualitative features are useful in understanding the nature of $X(3872)$ and should be

further tested and studied experimentally and theoretically.

In summary, we study the decays $B \rightarrow \chi_{c1}(1P, 2P)K$ in QCD factorization by treating charmonia as nonrelativistic bound states. We find that there are no infrared divergences in the vertex corrections, and the logarithmic end-point singularities from hard spectator interactions can be regularized by a momentum cutoff. Within certain uncertainties we find the $B \rightarrow \chi_{c1}(2P)K$ decay rate can be comparable to $B \rightarrow \chi_{c1}(1P)K$, and get $Br(B^0 \rightarrow \chi'_{c1}K^0) = Br(B^+ \rightarrow \chi'_{c1}K^+) \approx 2 \times 10^{-4}$. This might imply that the $X(3872)$ has a dominant $J^{PC} = 1^{++}(2P)$ $c\bar{c}$ component but mixed with some $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ continuum component. The qualitative features of $X(3872)$ are discussed and should be further tested and studied.

Note. After this work appeared in hep-ph/0506222 we learned some new results from BaBar [26]: $Br(B^+ \rightarrow X(3872)K^+) < 3.2 \times 10^{-4}$, $R = \frac{Br(B^0 \rightarrow X(3872)K^0)}{Br(B^+ \rightarrow X(3872)K^+)} = 0.50 \pm 0.30 \pm 0.05$. We also note that a recent paper[27] (hep-ph/0508258) obtained similar conclusions to ours for the $X(3872)$.

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